

Math 3510H — Assignment #1:

Due Wednesday, October 7, 2020 (submitted by 4pm via Blackboard in PDF format)

1. (a) A debt of \$2000 due in 3 years and \$5000 due in 9 years is to be repaid by a single payment in four years. If the interest rate is constant at 2% per annum (compounded continuously), what will be the value of the single payment?

(b) If you invest \$1000 at 10% (compounded annually), how long will it take for your investment to triple in value?

(c) The Rule of 72 gives an estimate for the doubling time of money. Derive a similar formula that estimates the tripling time of an investment at at given rate of r percent (compounded annually).

2. Weather forecaster Gonnarain is so certain it will rain on Sunday that he would offer a bet of 3:2 that it will happen. Remember, if it rains after someone bets with Gonnarain, Gonnarain keeps all of the money. If it does not rain, then Gonnarain pays the better \$3 for each \$2 that was wagered. On a different channel, Alwaysunny is confident that on Sunday it will not rain; she offers 4:3 odds.

(a) A person has \$100. Set up a hedge where the person will win the same amount of money no matter what happens on Sunday.

(b) Draw the two profit lines.

(c) How much money is the person guaranteed to win no matter what happens?

(d) Both forecasters believe they are making a fair bet; that is, they are willing to take either side. Determine the likelihood each has for rain on Sunday, for no rain on Sunday.

(e) Add the probabilities of rain from Alwaysunny and of no rain from Gonnarain. Add the probabilities of rain from Gonnarain and no rain from Alwaysunny. What do you notice? How does the answer help you understand when a bet is to your advantage?

3. (a) Consider a forward contract that matures in 3 months on an asset that has price \$40, and assume interest rates are 5% (compounded

continuously). Construct portfolios that provide guaranteed profits when the:

- (i) Forward price is \$43
- (ii) Forward price is \$39

(b) Use no-arbitrage arguments to derive a formula for the forward price F_0 of a forward contract (with maturity T) on a stock with spot price S_0 , if the interest rate is r and the stock has a dividend yield of q .

4. A long butterfly spread with calls is created by buying one call at a lower strike price, selling two calls with a higher strike price and buying one call with an even higher strike price. All calls have the same expiration date, and the strike prices are equidistant.

(a) Graph the profit curve for a butterfly spread created from European call options with strike prices 95, 100 and 105. When is a butterfly spread profitable? Describe what type of hedging protection it provides.

(b) Consider a butterfly spread created with European call options with strike prices K_1 , K_2 , and K_3 , where $K_1 \leq K_2 \leq K_3$ and respective prices C_1 , C_2 , and C_3 . Prove that:

$$C_2 \leq \frac{C_1 + C_3}{2}.$$